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THERMAL EFFECTS IN SHOCKS IN VISCOPLASTIC SOLIDS

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(*Received* 13*th July* 1970)

Summary

In previous work by the present writers, a system of equations was developed governing finite one-dimensional strain waves in a strain-rate sensitive elastic viscoplastic material, and in the present paper these equations are extended to include the effects of heating caused by compression and plastic dissipation. By assuming adiabatic conditions, a non-linear integral equation is established for the temperature increase. This equation is simplified by using the previously-discussed approximation of steady-state propagation of the wave and a nearly exact solution is given for this form of the equation. As a result, it is shown that the temperature can be expressed as a function of the density only in a steady-state wave and is independent of the plastic strain-rate relation used to describe the material. On the basis of the analysis, examples are given showing temperature changes and other features of shock waves in rate-sensitive plastic solids.

1. Introduction

As a technique for the study of the rate-sensitive plastic behaviour of metals at high rates-of-strain the plane shock wave experiment has certain potential advantages associated with the level of strain rate which can be induced and the commonly accepted belief that no geometrical dispersion effects occur. It has frequently provided the motivation for the construction of material constitutive relations and has been the principal means for determining material parameters for some of these relations.

In the earliest plane wave experiments two parameters that could be determined were the Hugoniot elastic limit (or stress level associated with the elastic precursor wave) and the dynamic compressibility (or bulk modulus) associated with the following plastic wave (or shock wave in the terminology used here). A comprehensive review of relevant theoretical and experimental work prior to 1957 is given by Rice, McQueen and Walsh (1958) and subsequent research along these lines has been reviewed by Duvall (1964). More complex theories of material behaviour have led to the extraction of much additional information from such experiments. The analysis of precursor-amplitude decay with specimen thickness has been suggested by several investigators (Taylor, 1965, Johnson and Band, 1967, Kelly and Gillis,

1967) as a fruitful technique for the evaluation of proposed constitutive equations. A significant development in this area is due to ROHDE (1969) who seems to have made the first systematic study of precursor-amplitude dependence upon temperature. Rohde then attempted to reconcile several postulated material equations with his experimental results.

More recently, KELLY and GILLIS (1970) (subsequently denoted by (KG)), proposed that evaluations could also be made from the study of shock thicknesses. We developed a system of equations governing the kinematics of propagation of one-dimensional strain waves of finite deformation in an elastic viscoplastic material. A simple dynamical condition was obtained by assuming steady-state propagation of the wave. Under this assumption a method was presented which allows the principal features of the wave to be calculated with great accuracy by elementary numerical techniques.

Two limitations restrict the applicability of this previous analysis. In some physical situations the steady-state assumption is not a useful approximation to reality and in these cases the dynamical condition is inadequate. In other cases thermal effects can not be neglected as was done previously in the kinematic analysis. The purpose of the present paper is to account in the kinematical equations for irreversible entropy production and effects of temperature on the rate sensitivity of the material constitutive equation. This will extend the applicability of the previous calculational method to waves in which substantial increases of temperature occur.

An extended kinematical formulation of this sort has been presented by KELLY (1970) in very general form. We will be concerned here with a less exact but more easily usable extension. By approximating the entropy production integral, we obtain a simple expression for the dependence of temperature upon density and this allows the previous computational procedure to be employed with only slight modifications. Thus, the largest stresses, deformation rates, shock thickness and other features of the wave can be calculated without recourse to direct integration of the wave equation, a process that is frequently beset by computational difficulties and is limited in accuracy by several factors, the most prominent of which is the integration technique.

2. THEORY

Since the main kinematical arguments were given in (KG), only the pertinent results from that analysis will be stated here. The critical reader will refer to (KG) for the full details of these earlier derivations. We keep the definitions for principal strains ε_i as the natural logarithms of their respective stretch ratios λ_i, dilatation $e = \varepsilon_1+\varepsilon_2+\varepsilon_3$, deviatoric strains $e_i = \varepsilon_i - e/3$, and use superscripts '$e$' and '$p$' to denote 'elastic' and 'plastic' respectively.

As in the construction of a linear theory of thermoelasticity (FUNG, 1965) we assume the existence of a Helmholtz free energy function F which is a function of the dilatation e, the deviatoric elastic strains e_i^e and the temperature T, having the form

$$\rho_0 F = \tfrac{1}{2}K(e^e)^2 + G[(e_1^e)^2+(e_2^e)^2+(e_3^e)^2] - K\beta(T-T_0)e - g. \tag{1}$$

Here, ρ_0 is the initial mass density, K and G the bulk and shear moduli, β the volumetric coefficient of thermal expansion, T_0 the temperature of the reference con-